



Medida de ángulos	
$2\pi \text{ rad} \rightarrow 360^\circ \rightarrow 400^\circ$	
<b>Radianes a grados</b>	<b>Grados a radianes</b>
$\alpha(\text{rad}) = \frac{2\pi}{360} \alpha(^{\circ})$	$\alpha(\text{rad}) = \frac{2\pi}{400} \alpha(^{\circ})$
<b>Longitud de arco (Ángulo en radianes)</b>	
$L = R \cdot \alpha$	

Relaciones fundamentales	
<b>Inversas</b>	<b>Fundamentales</b>
$\sec \alpha = \frac{1}{\cos \alpha}$	$\text{tg } \alpha = \frac{\text{sen } \alpha}{\cos \alpha}$
$\text{cosec } \alpha = \frac{1}{\text{sen } \alpha}$	$\text{sen}^2 \alpha + \text{cos}^2 \alpha = 1$
$\text{cotg } \alpha = \frac{1}{\text{tg } \alpha}$	$\text{tg}^2 \alpha + 1 = \text{sec}^2 \alpha$
	$1 + \text{cotg}^2 \alpha = \text{cosec}^2 \alpha$

Ángulos relacionados	
<b>Ángulos complementarios (<math>\alpha + \beta = 90^\circ</math>)</b>	
$\text{sen } \alpha = \cos(90^\circ - \alpha)$	$\text{sen}(90^\circ - \alpha) = \cos \alpha$
$\text{tg } \alpha = \text{cotg}(90^\circ - \alpha)$	$\text{tg}(90^\circ - \alpha) = \text{cotg } \alpha$
$\text{sec } \alpha = \text{cosec}(90^\circ - \alpha)$	$\text{sec}(90^\circ - \alpha) = \text{cosec } \alpha$

<b>Ángulos suplementarios (<math>\alpha + \beta = 180^\circ</math>)</b>	
$\text{sen}(180^\circ - \alpha) = \text{sen } \alpha$	
$\text{cos}(180^\circ - \alpha) = -\text{cos } \alpha$	

<b>Ángulos opuestos</b>	
$\text{sen } \alpha = -\text{sen}(-\alpha)$	
$\text{cos } \alpha = \text{cos}(-\alpha)$	

<b>Ángulos que difieren en <math>180^\circ</math></b>	
$\text{sen}(180^\circ + \alpha) = -\text{sen } \alpha$	
$\text{cos}(180^\circ + \alpha) = -\text{cos } \alpha$	

<b>Ángulos que difieren en <math>90^\circ</math></b>	
$\text{sen}(90^\circ + \alpha) = \text{cos } \alpha$	
$\text{cos}(90^\circ + \alpha) = -\text{sen } \alpha$	

Razones trigonometricas de un ángulo agudo	
<b>Funciones</b>	<b>Cofunciones</b>
$\text{sen } \alpha = \frac{\text{cat. opuesto}}{\text{hipotenusa}} = \frac{a}{c}$	$\text{cos } \alpha = \frac{\text{cat. contiguo}}{\text{hipotenusa}} = \frac{b}{c}$
$\text{tg } \alpha = \frac{\text{cat. opuesto}}{\text{cat. contiguo}} = \frac{a}{b}$	$\text{cotg } \alpha = \frac{\text{cat. contiguo}}{\text{cat. opuesto}} = \frac{b}{a}$
$\text{sec } \alpha = \frac{\text{hipotenusa}}{\text{cat. contiguo}} = \frac{c}{b}$	$\text{cosec } \alpha = \frac{\text{hipotenusa}}{\text{cat. opuesto}} = \frac{c}{a}$

Ángulos notables del 1º cuadrante					
	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\text{sen } \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\text{cos } \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\text{tg } \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\infty$

<b>Resolución de triángulos cualesquiera</b>	
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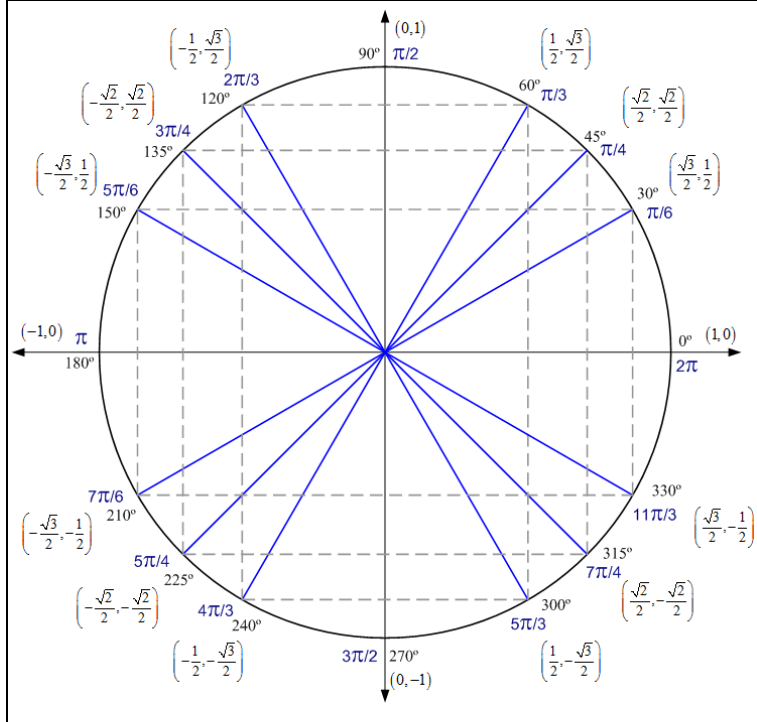
<b>Teorema del seno</b>
$\frac{a}{\text{sen } A} = \frac{b}{\text{sen } B} = \frac{c}{\text{sen } C}$

<b>Teorema del coseno</b>
$a^2 = b^2 + c^2 - 2bc \cdot \text{cos } A$
$b^2 = a^2 + c^2 - 2ac \cdot \text{cos } B$
$c^2 = a^2 + b^2 - 2ab \cdot \text{cos } C$

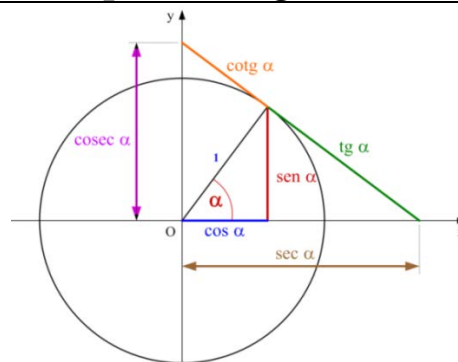
<b>Teorema de la tangente</b>
$\frac{a-b}{a+b} = \frac{\text{tg}\left(\frac{A-B}{2}\right)}{\text{tg}\left(\frac{A+B}{2}\right)}$



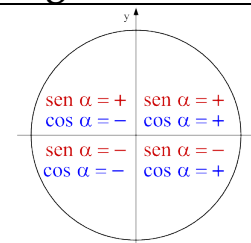
## Circunferencia Goniométrica



## Interpretación geométrica



## Signo de las razones trigonométricas



## Razones trigonométricas de la suma

$$\begin{aligned}\text{sen}(\alpha + \beta) &= \text{sen } \alpha \cdot \text{cos } \beta + \text{sen } \beta \cdot \text{cos } \alpha \\ \text{cos}(\alpha + \beta) &= \text{cos } \alpha \cdot \text{cos } \beta - \text{sen } \alpha \cdot \text{sen } \beta \\ \text{tg}(\alpha + \beta) &= \frac{\text{tg } \alpha + \text{tg } \beta}{1 - \text{tg } \alpha \cdot \text{tg } \beta}\end{aligned}$$

## Razones del ángulo doble

$$\begin{aligned}\text{sen}(2\alpha) &= 2 \text{sen } \alpha \cdot \text{cos } \alpha \\ \text{cos}(2\alpha) &= \text{cos}^2 \alpha - \text{sen}^2 \alpha \\ \text{tg}(2\alpha) &= \frac{2 \text{tg } \alpha}{1 - \text{tg}^2 \alpha}\end{aligned}$$

## Razones trigonométricas de la diferencia

$$\begin{aligned}\text{sen}(\alpha - \beta) &= \text{sen } \alpha \cdot \text{cos } \beta - \text{sen } \beta \cdot \text{cos } \alpha \\ \text{cos}(\alpha - \beta) &= \text{cos } \alpha \cdot \text{cos } \beta + \text{sen } \alpha \cdot \text{sen } \beta \\ \text{tg}(\alpha - \beta) &= \frac{\text{tg } \alpha - \text{tg } \beta}{1 + \text{tg } \alpha \cdot \text{tg } \beta}\end{aligned}$$

## Razones del ángulo mitad

$$\begin{aligned}\text{sen}\left(\frac{\alpha}{2}\right) &= \pm \sqrt{\frac{1 - \text{cos } \alpha}{2}} \\ \text{cos}\left(\frac{\alpha}{2}\right) &= \pm \sqrt{\frac{1 + \text{cos } \alpha}{2}} \\ \text{tg}\left(\frac{\alpha}{2}\right) &= \sqrt{\frac{1 - \text{cos } \alpha}{1 + \text{cos } \alpha}}\end{aligned}$$

## Transformación de sumas a productos

$$\begin{aligned}\text{sen } A + \text{sen } B &= 2 \text{sen}\left(\frac{A+B}{2}\right) \text{cos}\left(\frac{A-B}{2}\right) \\ \text{sen } A - \text{sen } B &= 2 \text{cos}\left(\frac{A+B}{2}\right) \text{sen}\left(\frac{A-B}{2}\right) \\ \text{cos } A + \text{cos } B &= 2 \text{cos}\left(\frac{A+B}{2}\right) \text{cos}\left(\frac{A-B}{2}\right) \\ \text{cos } A - \text{cos } B &= -2 \text{sen}\left(\frac{A+B}{2}\right) \text{sen}\left(\frac{A-B}{2}\right) \\ \text{sen}^2 \alpha - \text{sen}^2 \beta &= \text{sen}(\alpha + \beta) \text{sen}(\alpha - \beta) \\ \text{cos}^2 \alpha - \text{sen}^2 \beta &= \text{cos}(\alpha + \beta) \text{cos}(\alpha - \beta)\end{aligned}$$

## De productos a sumas

$$\begin{aligned}\text{sen } A \cdot \text{cos } B &= \frac{1}{2} [\text{sen}(A+B) + \text{sen}(A-B)] \\ \text{cos } A \cdot \text{sen } B &= \frac{1}{2} [\text{sen}(A+B) - \text{sen}(A-B)] \\ \text{cos } A \cdot \text{cos } B &= \frac{1}{2} [\text{cos}(A+B) + \text{cos}(A-B)] \\ \text{sen } A \cdot \text{sen } B &= -\frac{1}{2} [\text{cos}(A+B) - \text{cos}(A-B)]\end{aligned}$$